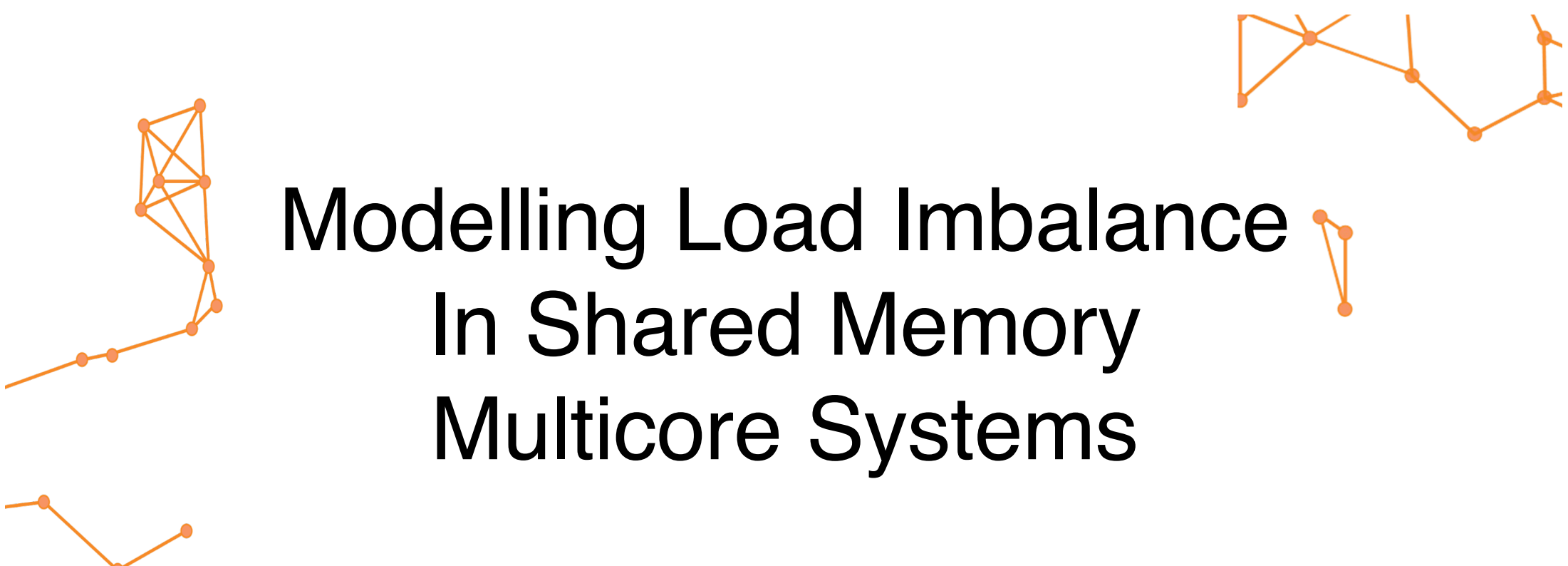


The slide features several decorative orange line graphs. On the left, there is a cluster of points connected by lines, resembling a network or a complex path. On the right, there is a series of points connected by lines, forming a zig-zag pattern. In the bottom left, there is a small cluster of points connected by lines, forming a simple path. In the bottom right, there is a small cluster of points connected by lines, forming a simple path.

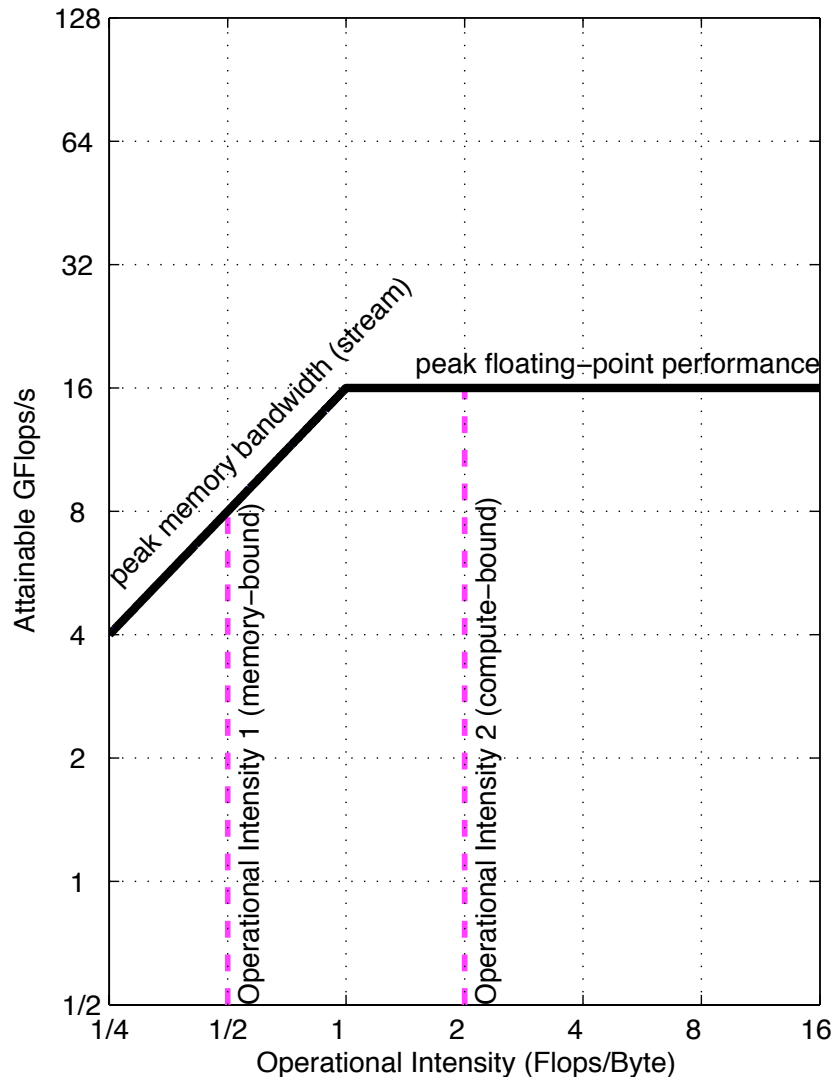
Modelling Load Imbalance In Shared Memory Multicore Systems

Johannes Langguth
joint work with Xing Cai and James Trotter



simula

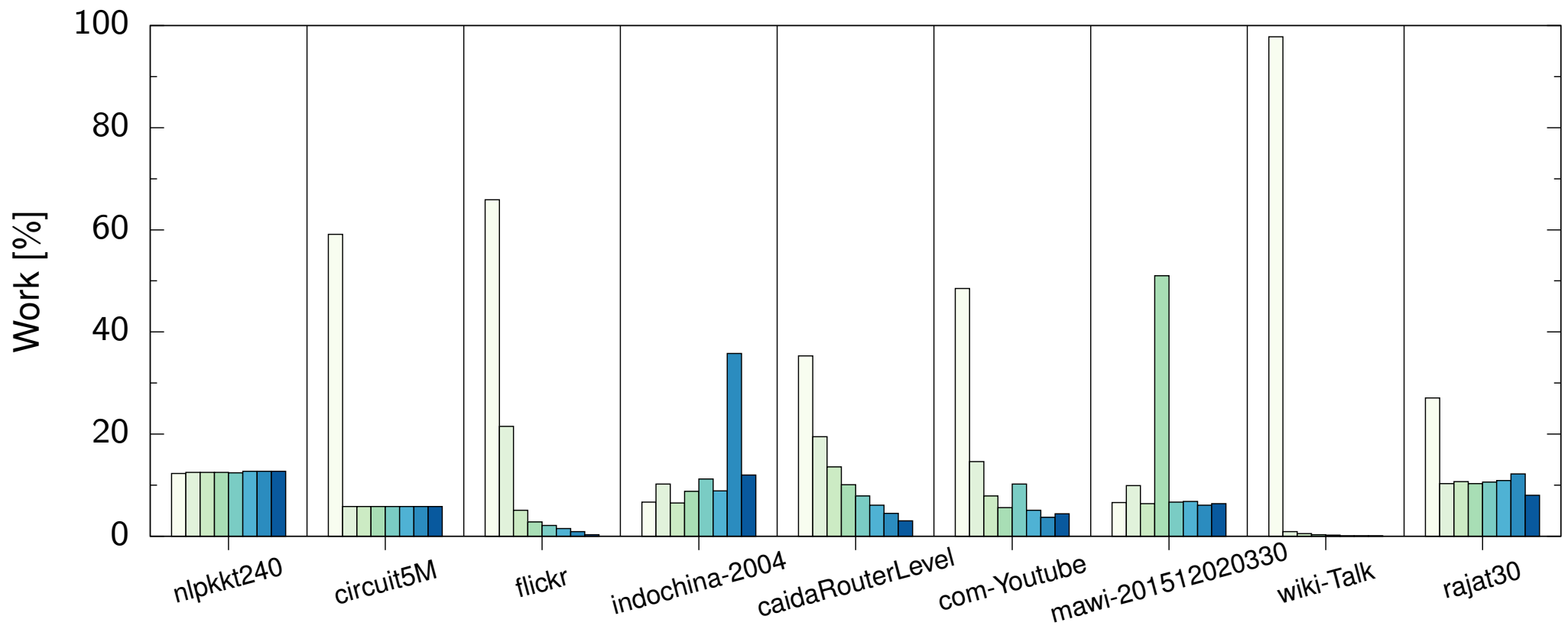
Performance Modelling for Multicore: the Roofline Model



- Widely accepted standard
- Not very accurate or detailed, but *very insightful*
- Performance prediction, but can easily be converted to time
- No special consideration of parallelism
- Works for parallel computation as long as load is balanced

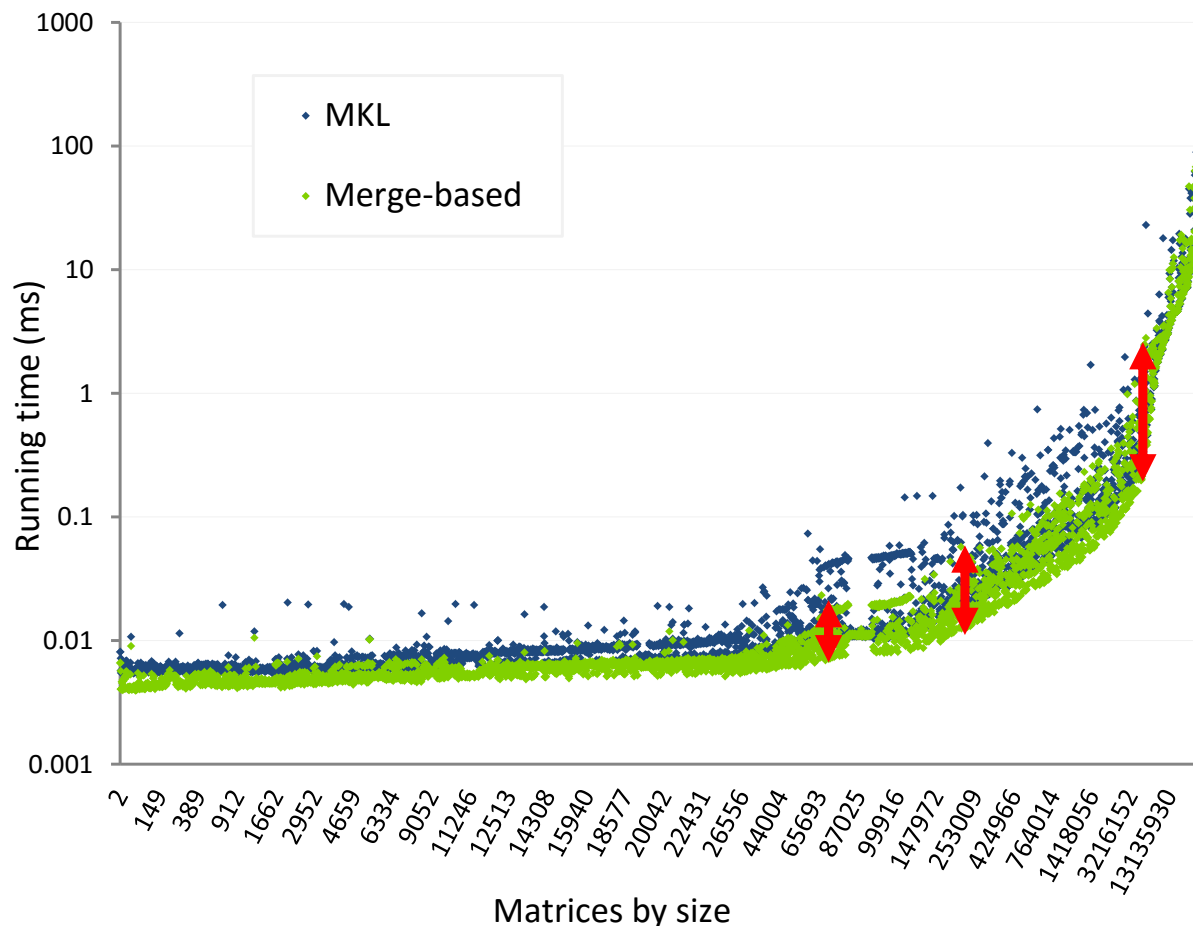
Williams et al., *Roofline: An Insightful Visual Performance Model for Floating-Point Programs and Multicore Architectures*, 2008

Common Problem: Load Imbalance



Number of nonzeros in 8-way even row split

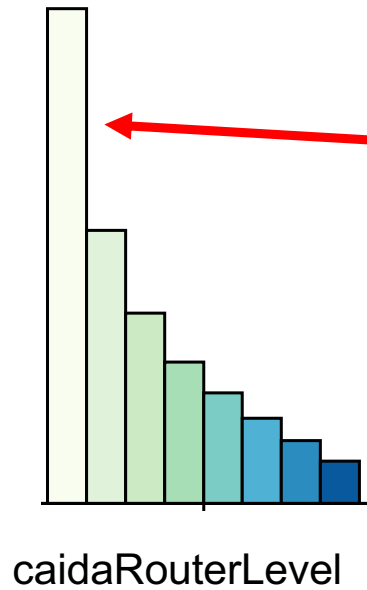
Hard to avoid Load Imbalance



- Even if memory access volume is balanced, actual work performed can vary
- Example here: caching behaviour in SpMV
- Many other examples

Duane Merrill and Michael Garland:
Merge-based parallel sparse matrix-vector multiplication, 2016

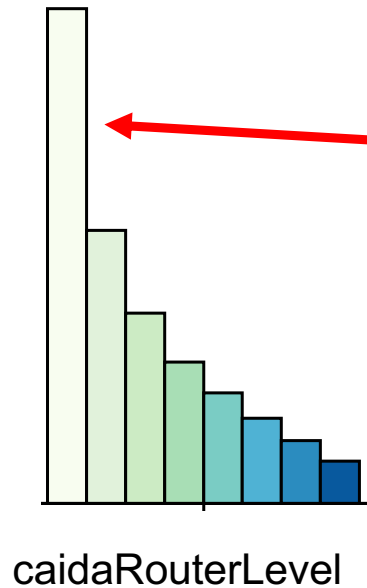
The Roofline Model under Load Imbalance



Obvious Idea:

Take the longest task and apply the model to a single core

How can we apply the Roofline Model here?

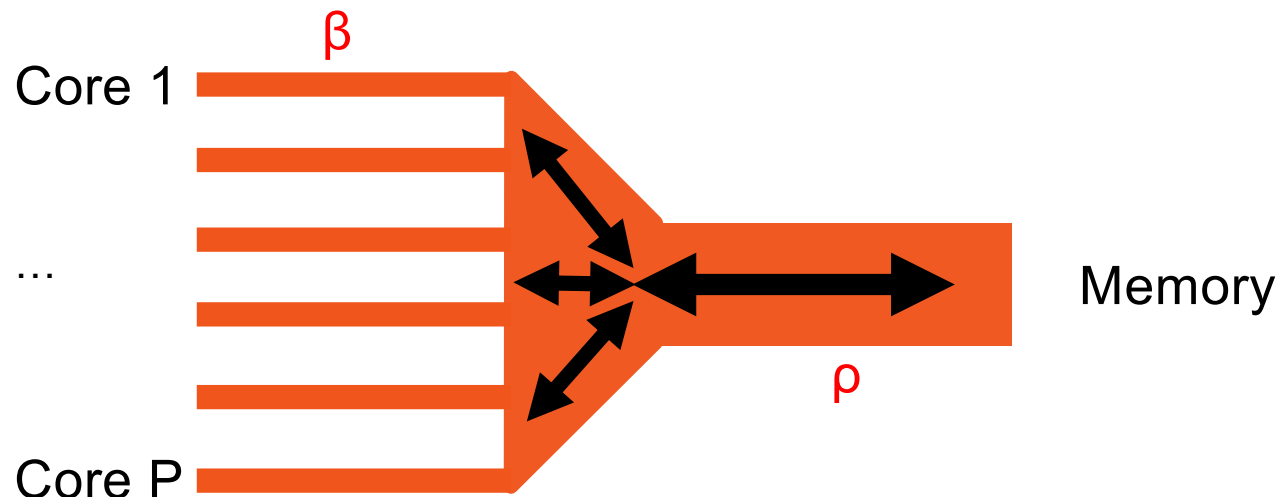


Obvious Idea:

Take the longest task and apply the model to a single core

- If compute bound, we are done (thus, no need to discuss arithmetic intensity here)
- If memory bound, what is the bandwidth for the longest task?

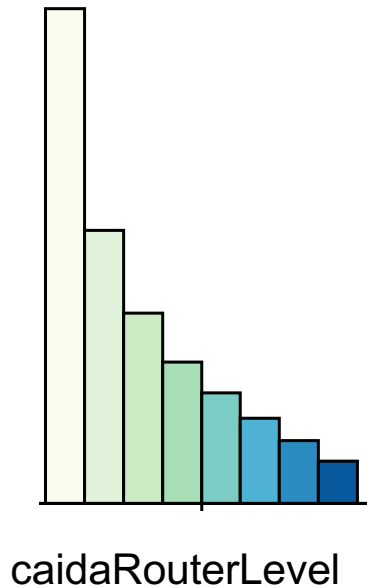
A simple model of Multicore Memory Bandwidth



- Effectively a Max-Rate model
- Works for multicore and NIC
- Assume $\beta < \rho < P\beta$

Gropp et al.: *Modeling MPI communication performance on SMP nodes: Is it time to retire the ping pong test*, 2016

How can we apply the Roofline Model here?

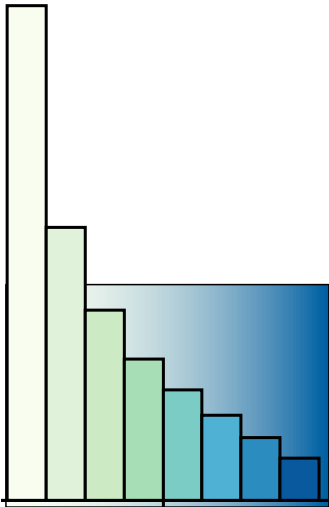


Obvious Idea:

Take the longest task and apply the model to a single core

- If compute bound, we are done
- If memory bound, what is the single core bandwidth?
 - A. Full Contention: ρ/P (all core STREAM by core count P)
 - B. No Contention: β (single core STREAM)

How can we apply the Roofline Model here?



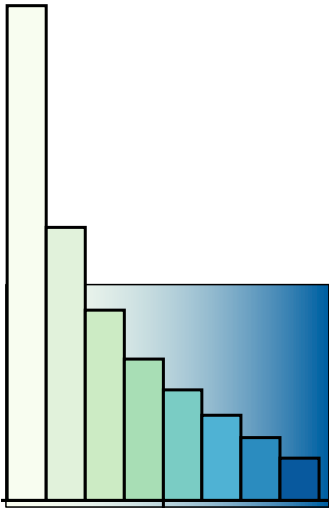
caidaRouterLevel

Another Idea (for the memory bound case):
Take the sum of all tasks and apply the model to all cores

For convenience:

- Let M_i be the workload of Processor i
- Sort processors in descending order of workload

How can we apply the Roofline Model here?



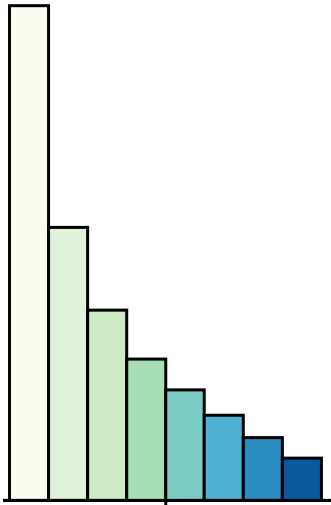
caidaRouterLevel

Another Idea (for the memory bound case):
Take the sum of all tasks and apply the model to all cores

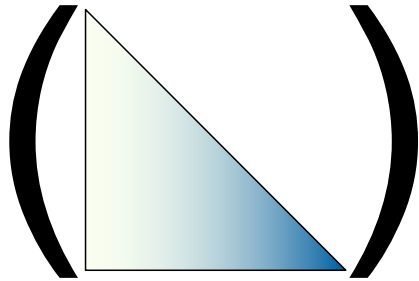
- A. *Full Contention:* $M_1 / (\rho/P)$
- B. *No Contention:* M_1 / β
- C. *No Imbalance:* $\sum_i M_i / \rho$

Let's test these Models

We need imbalanced workloads



caidaRouterLevel



triangular matrix

Triangular workload

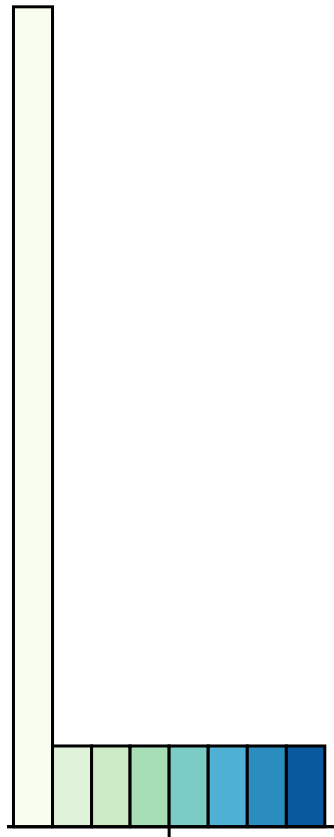
Inspired by dense triangular matrix

Test code:

Triangular matrix-vector multiply

```
#pragma omp parallel for
for (int i = 0; i < N; i++) {
    double z = 0.0;
    for (int j = 0; j < i; j++)
        z += A[i*N+j]*x[j];
    y[i] += z;
}
```

A second imbalanced Workload



circuit5M

Simple imbalanced workload
related to Amdahl's Law

$M_1 = (P+1)S$, $M_{i>1} = S$, for some value of S
(same work with and without contention)

Test code:

Imbalanced STREAM benchmark "imbaSTREAM"

Triangular Results on various CPUs

	Measured	Full Contention		No Contention		No Imbalance		Phase 1
CPU	[GB/s]	[GB/s]	Error	[GB/s]	Error	[GB/s]	Error	Time
AMD Epyc 7302P (Rome)	60.24	46.92	-22%	188.53	213%	90.91	51%	91.30%
AMD Epyc 7413 (Milan)	88.21	52.38	-41%	390.03	342%	102.58	16%	96.70%
AMD Epyc7601 (Naples)	66.9	43.39	-35%	295.02	341%	85.42	28%	96.40%
AMD Epyc7763 (Milan)	127.07	61.09	-52%	997.44	685%	121.23	-5%	99.40%
Intel Xeon Gold 6130	44.1	38.58	-13%	110.82	151%	74.74	69%	81.50%
Intel Xeon Platinum 8168	63.98	35.21	-45%	144.68	126%	68.96	8%	90.60%
Intel Xeon Platinum 8360Y	89.67	80.22	-11%	272.01	203%	158.21	76%	85.70%
Cavium Thunder X2	74.09	60.21	-19%	252.12	240%	118.54	60%	90.40%
HiSilicon Kunpeng 920	92.46	66.29	-28%	398.32	331%	131.54	42%	95.00%
NVIDIA Grace GH200	273.57	159.34	-42%	984.78	260%	316.45	16%	95.30%

Results use the bandwidth
versions of the models

Mostly OK

Terrible

Mostly OK

simula

Amdahl Results on various CPUs

	Measured	Full Contention		No Contention		No Imbalance	
CPU	[GB/s]	[GB/s]	Error	[GB/s]	Error	[GB/s]	Error
AMD Epyc 7302P (Rome)	33.07	10.69	-67.66%	42.97	+29.94%	90.91	+174.89%
AMD Epyc 7413 (Milan)	43.52	8.21	-81.15%	61.10	+40.40%	102.58	+135.69%
AMD Epyc7601 (Naples)	22.46	5.18	-76.95%	35.20	+56.72%	85.42	+280.30%
AMD Epyc7763 (Milan)	47.34	3.73	-92.12%	60.90	+28.65%	121.23	+156.09%
Intel Xeon Gold 6130	22.07	8.79	-60.16%	25.26	+14.46%	74.74	+238.64%
Intel Xeon Platinum 8168	20.75	5.52	-73.41%	22.67	+9.25%	68.96	+232.36%
Intel Xeon Platinum 8360Y	28.14	8.55	-69.61%	29.00	+3.03%	158.21	+462.14%
Cavium Thunder X2	26.90	7.18	-73.29%	30.08	+11.84%	118.54	+340.71%
HiSilicon Kunpeng 920	22.21	4.05	-81.78%	24.32	+9.50%	131.54	+492.29%
NVIDIA Grace GH200	51.84	8.67	-83.28%	53.58	+3.36%	316.45	+510.42%

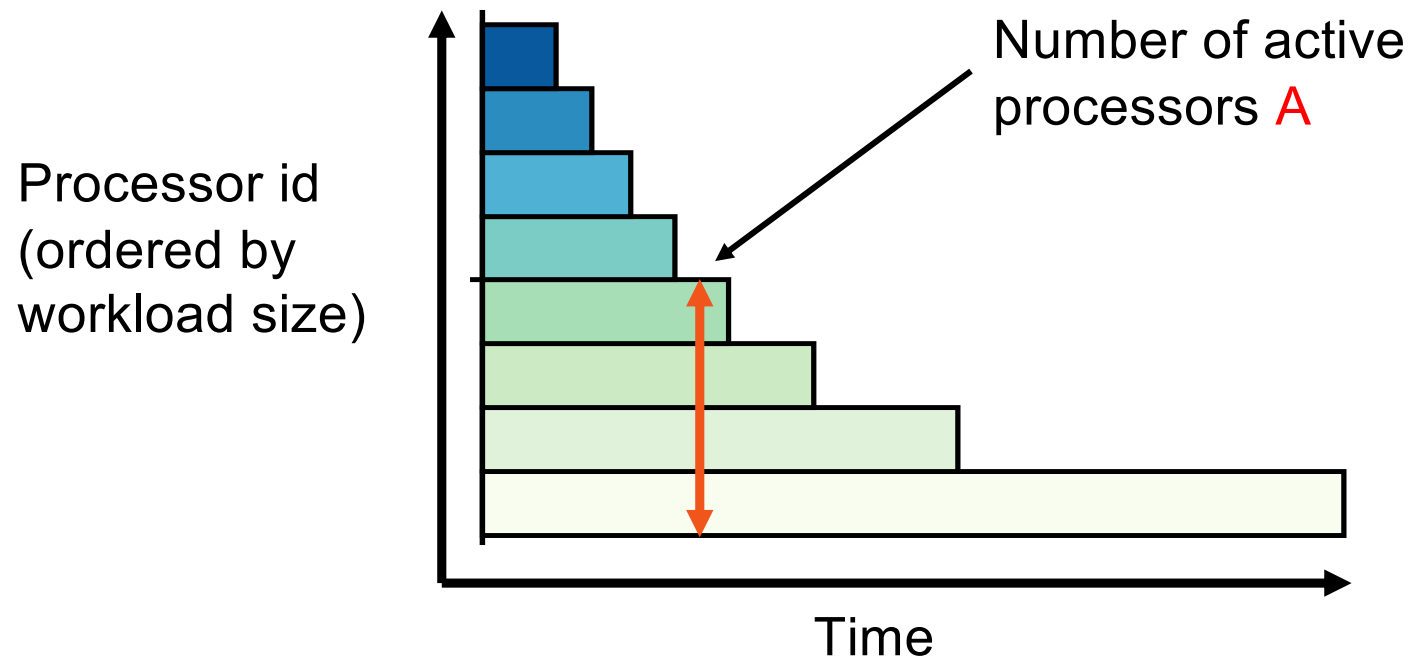
Terrible

Quite good

Terrible

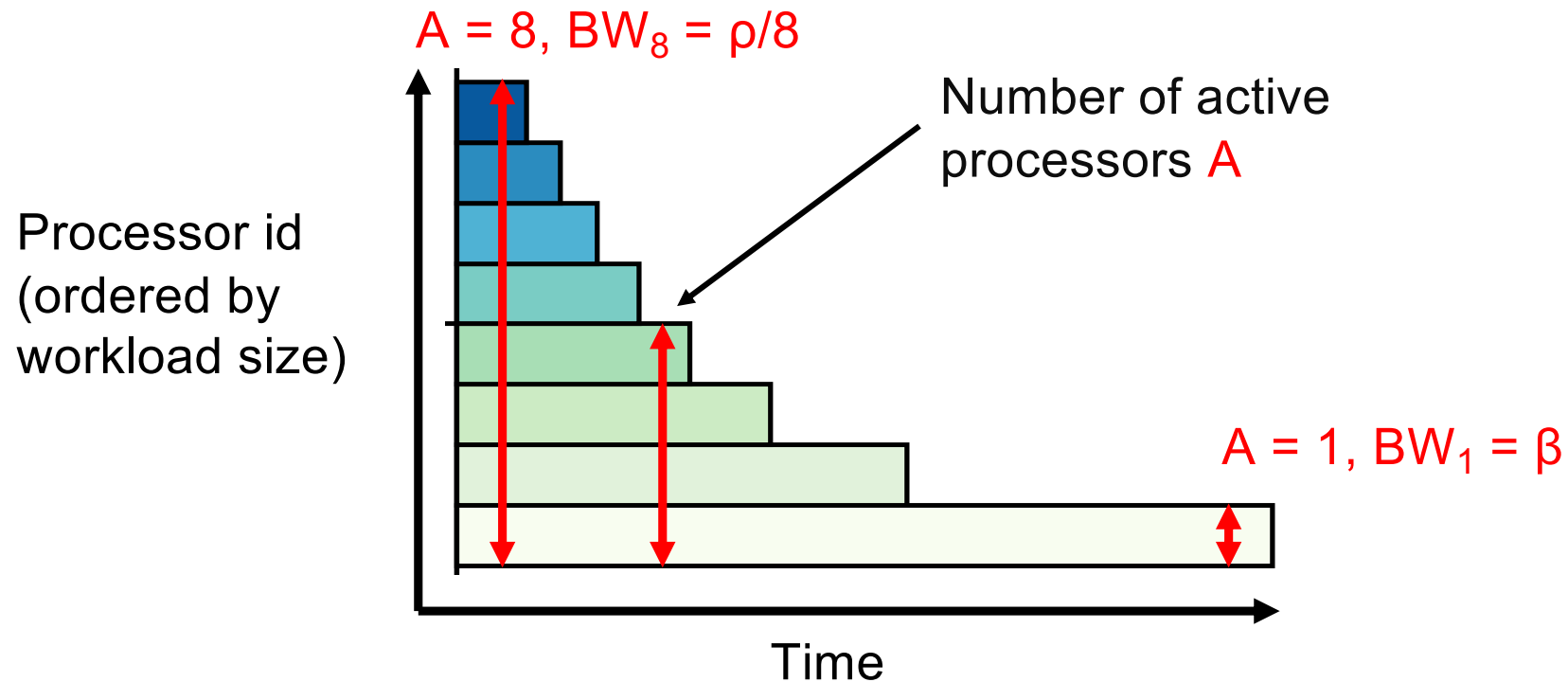
Results use the bandwidth
versions of the models

Why are the Models so Inaccurate?

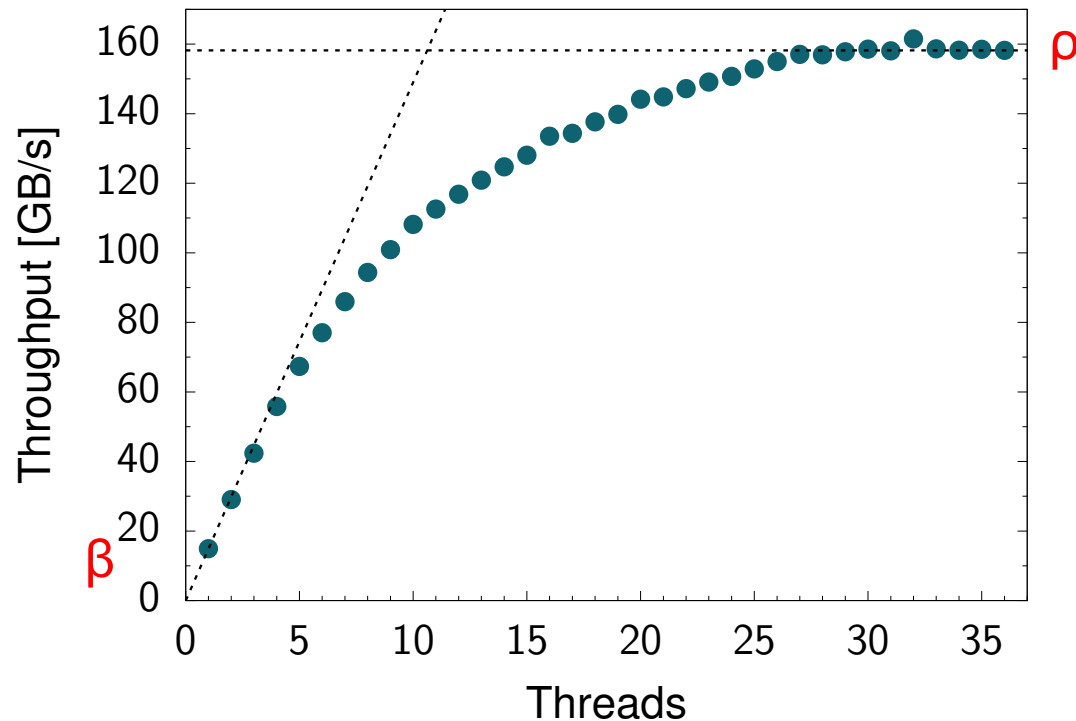


Processor activity over time

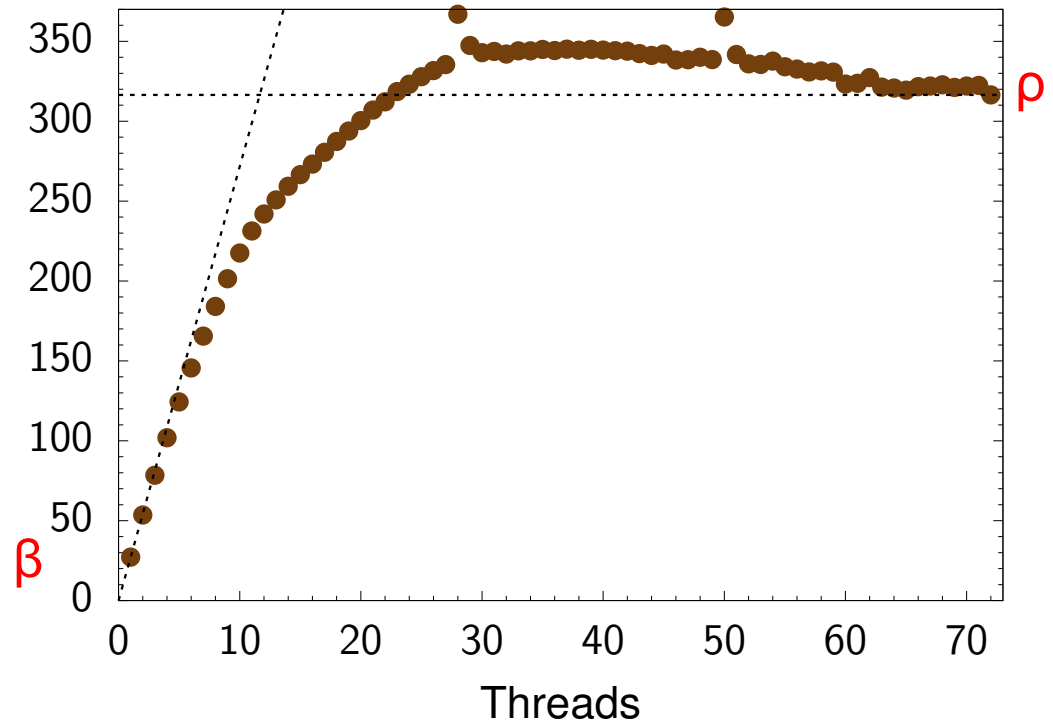
Performance changes during the computation



Accurate Modelling: need STREAM by #active cores



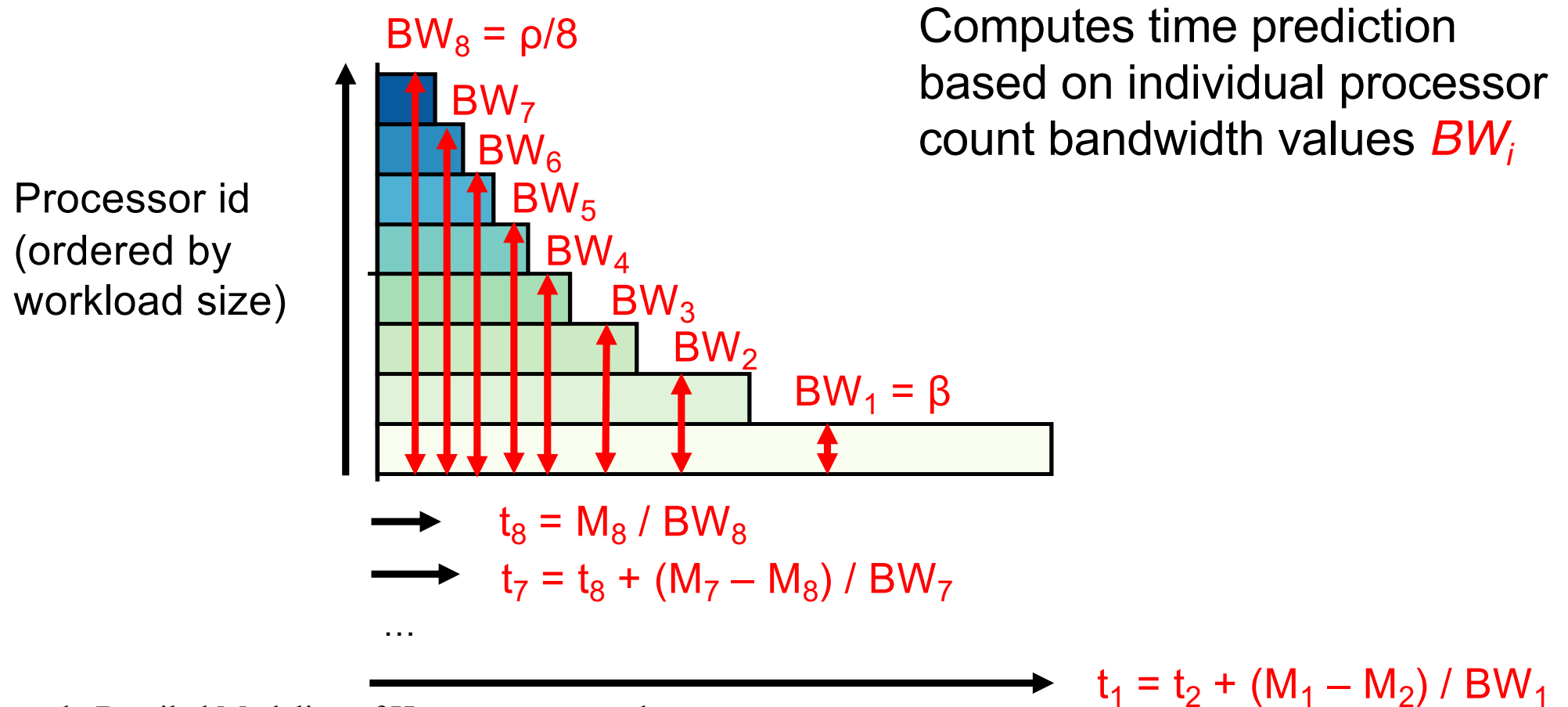
Intel Xeon Platinum 8360Y (Ice Lake)



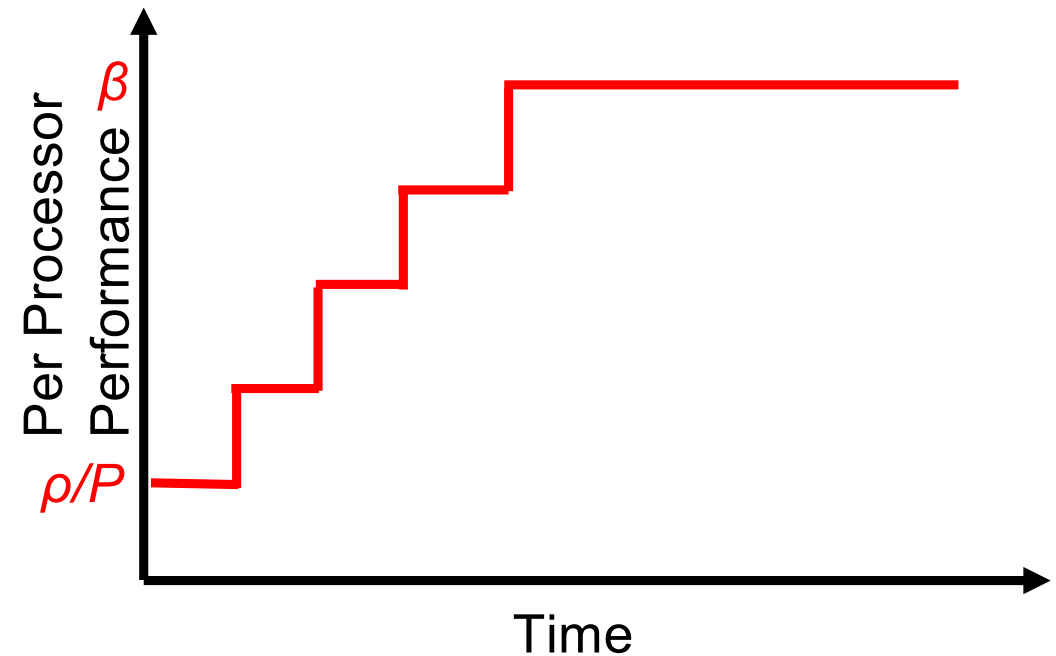
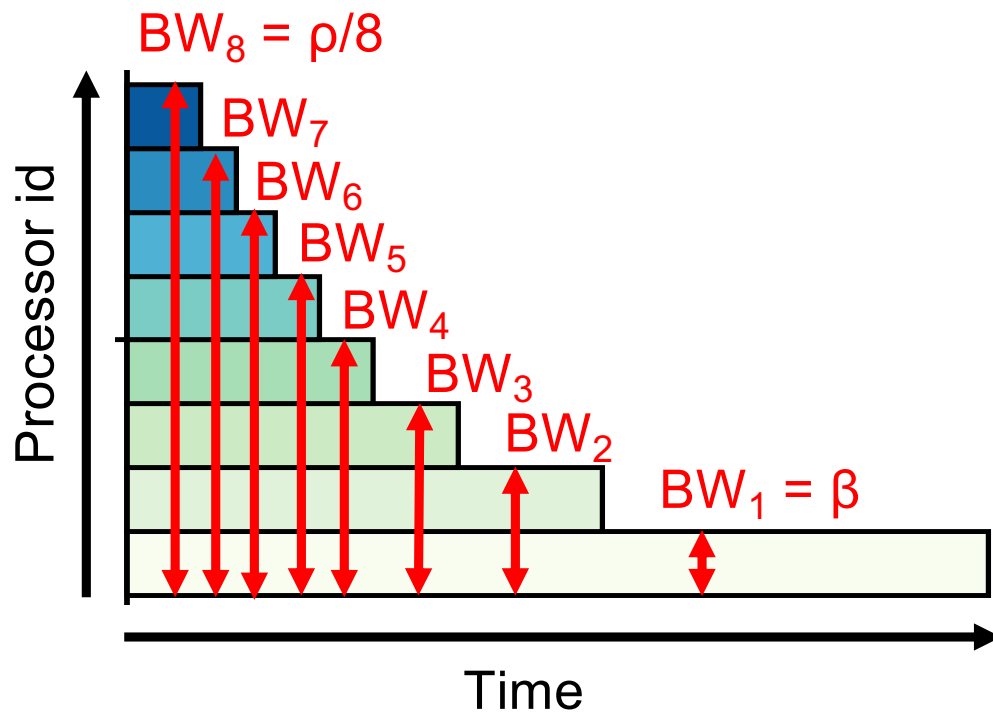
NVIDIA Grace GH200 CPU

Set of measurements BW_i = bandwidth with i active cores

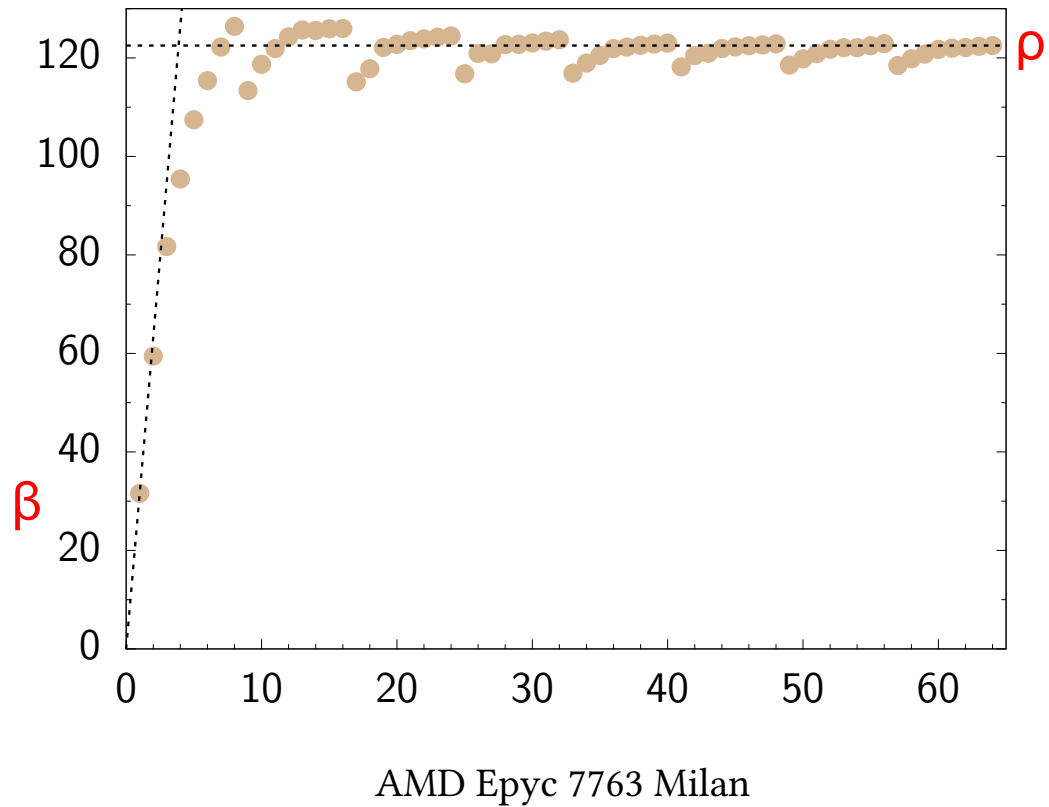
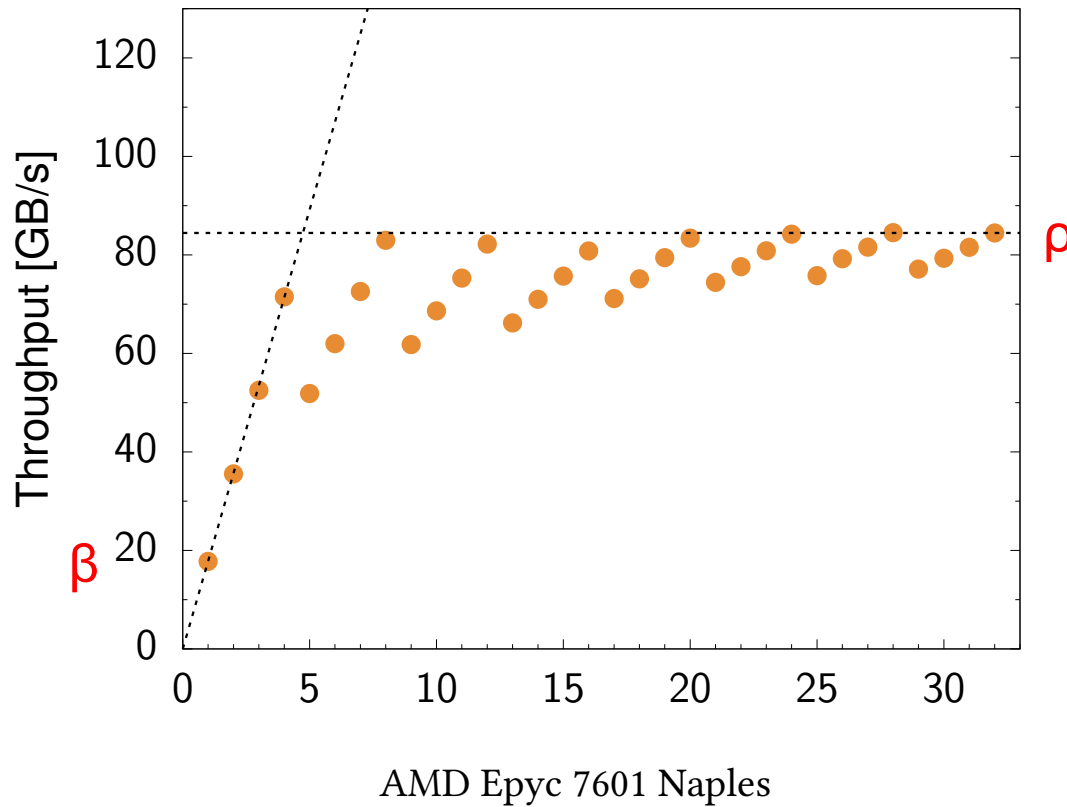
The Staircase Model



Performance per Core over Time



Problems with the Staircase Model

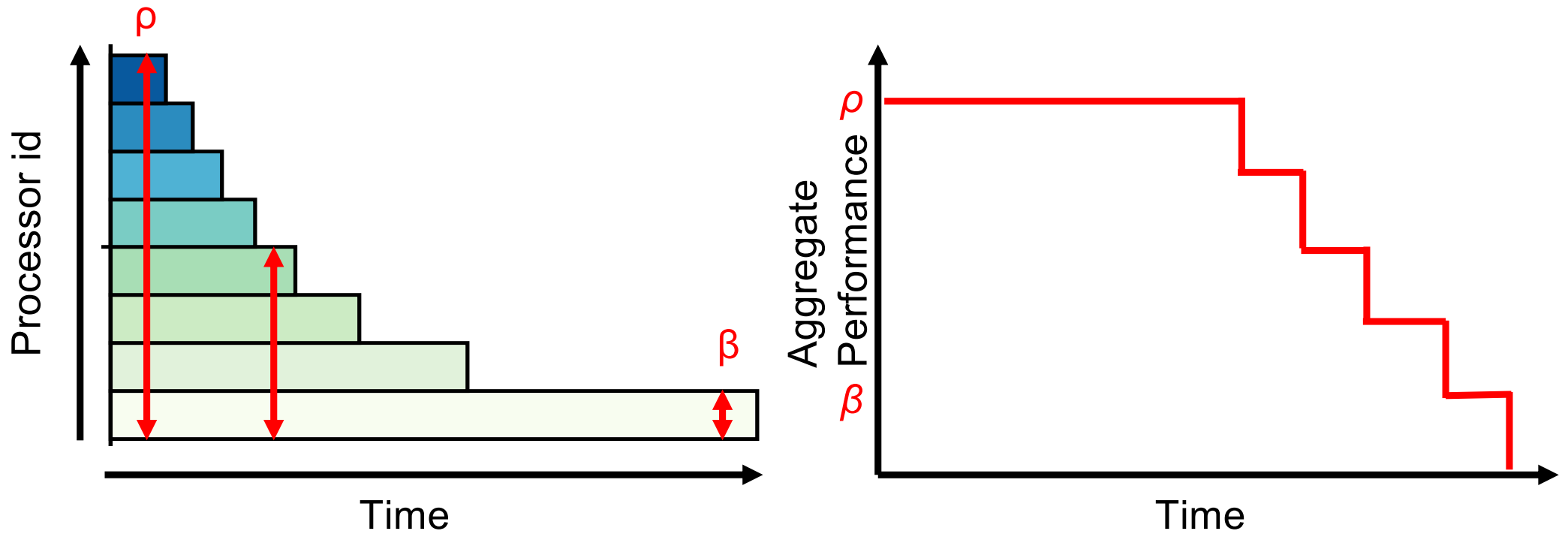


- BW_i is not a unique value, can depend on exact cores
- Model is very complex, not insightful

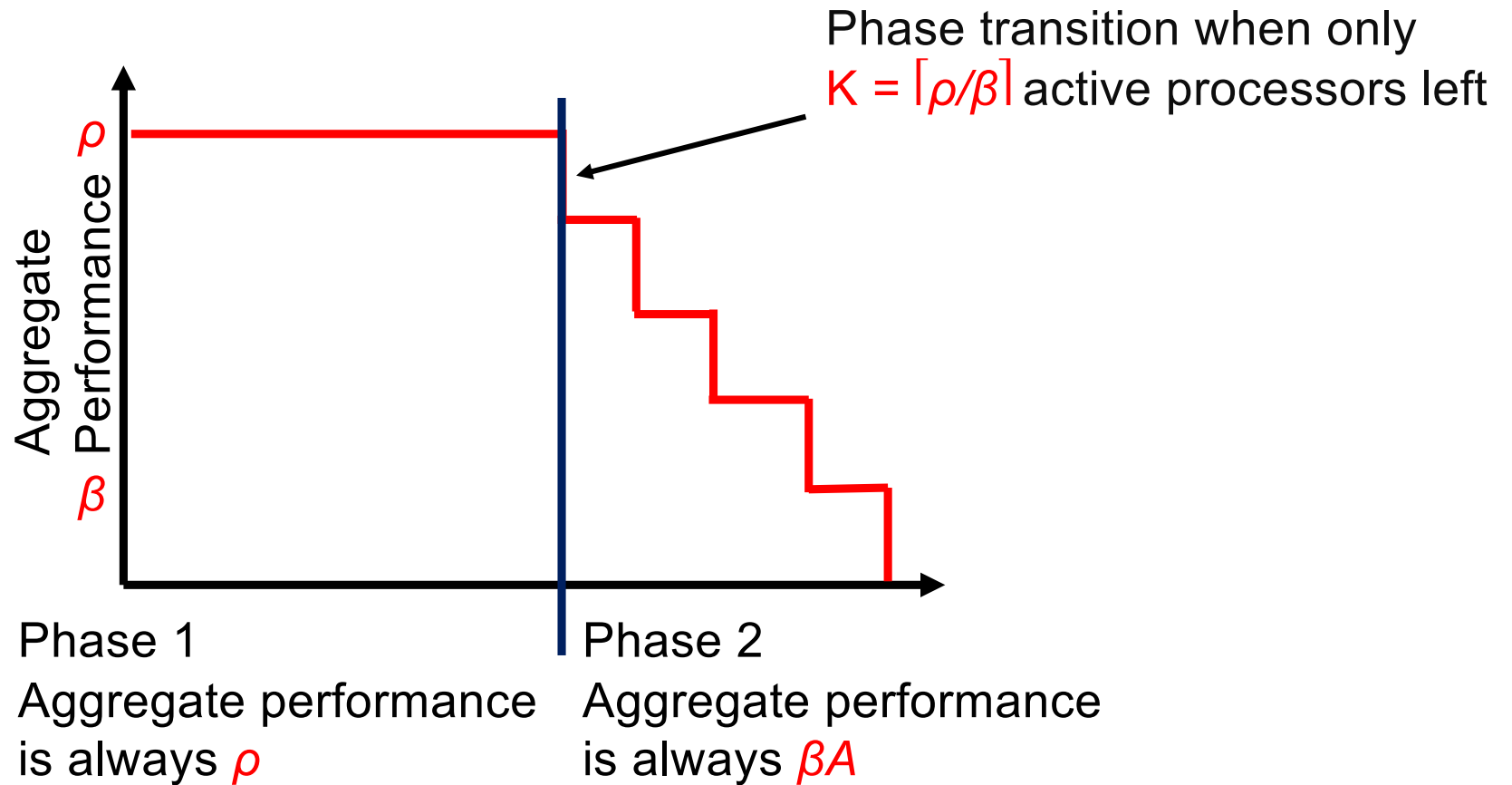
20

simula

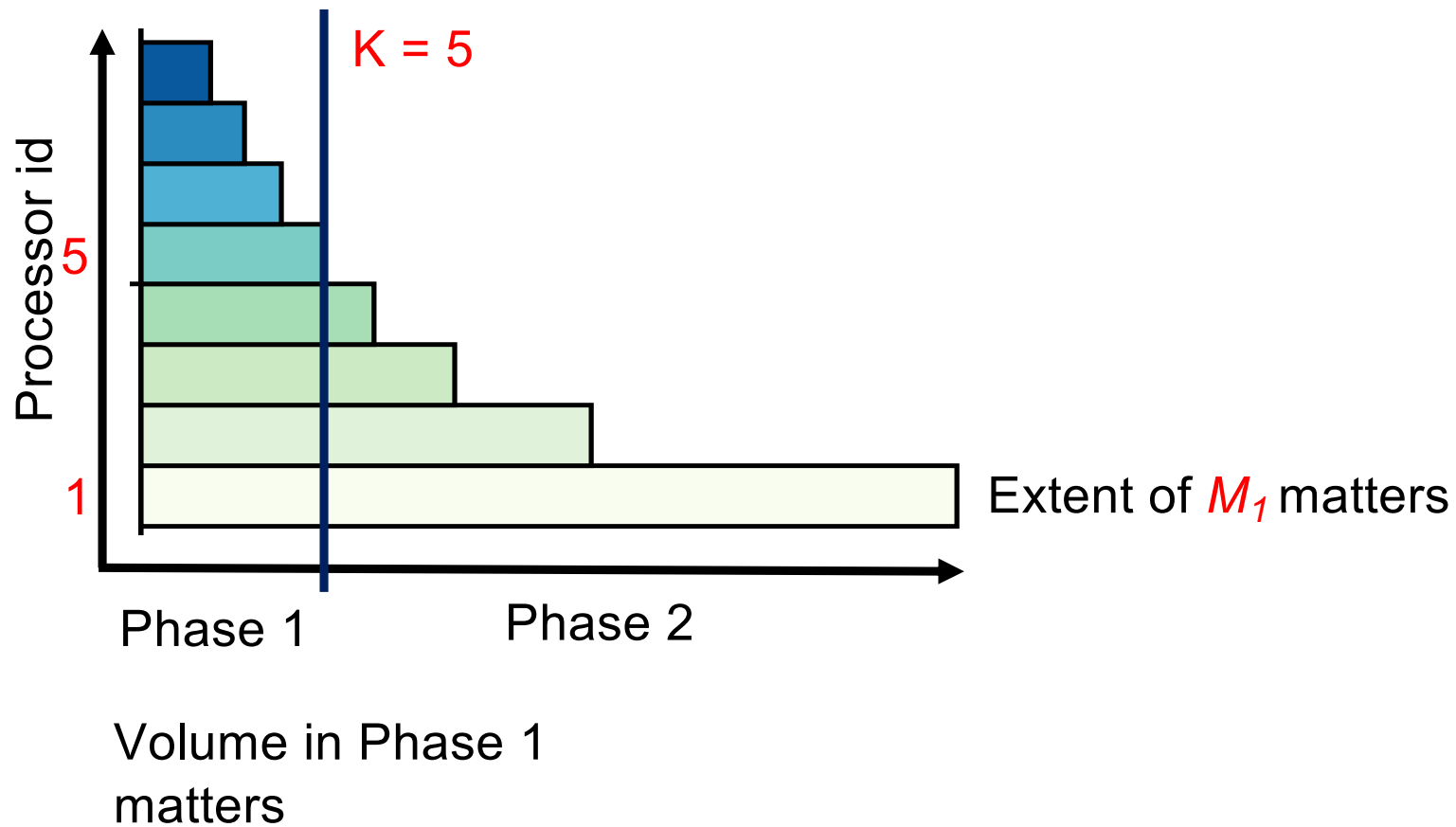
How can we do better? Aggregate Performance!



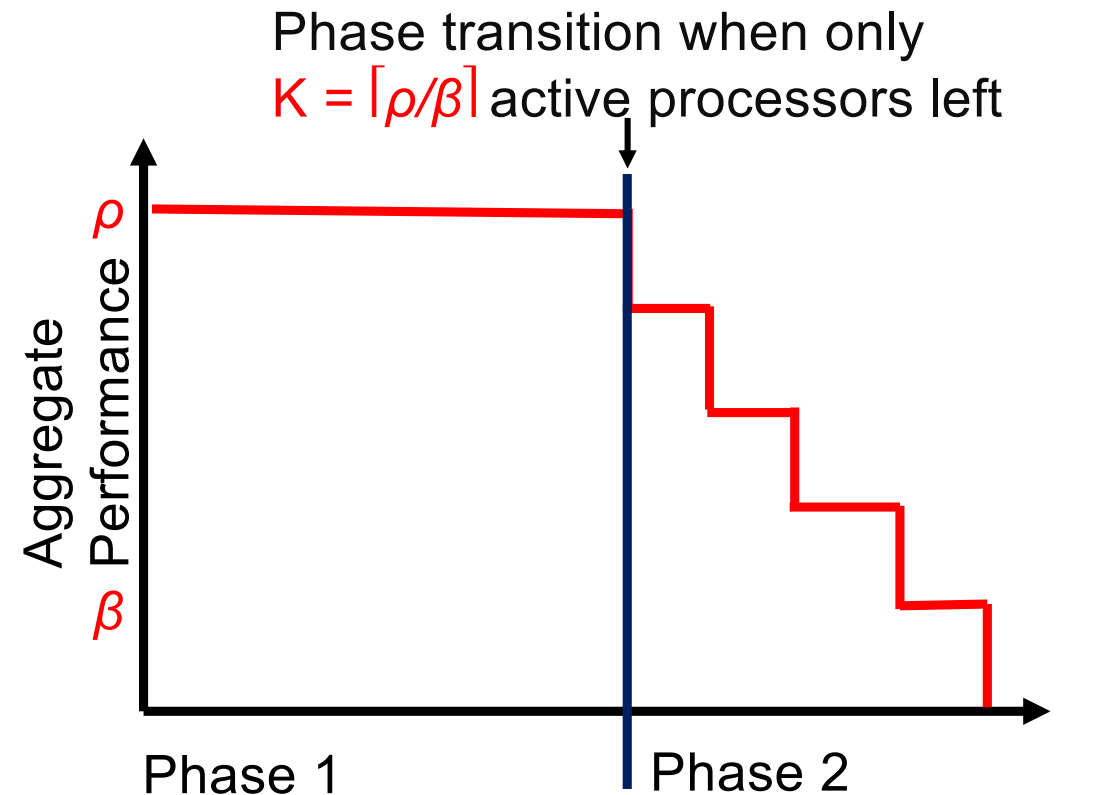
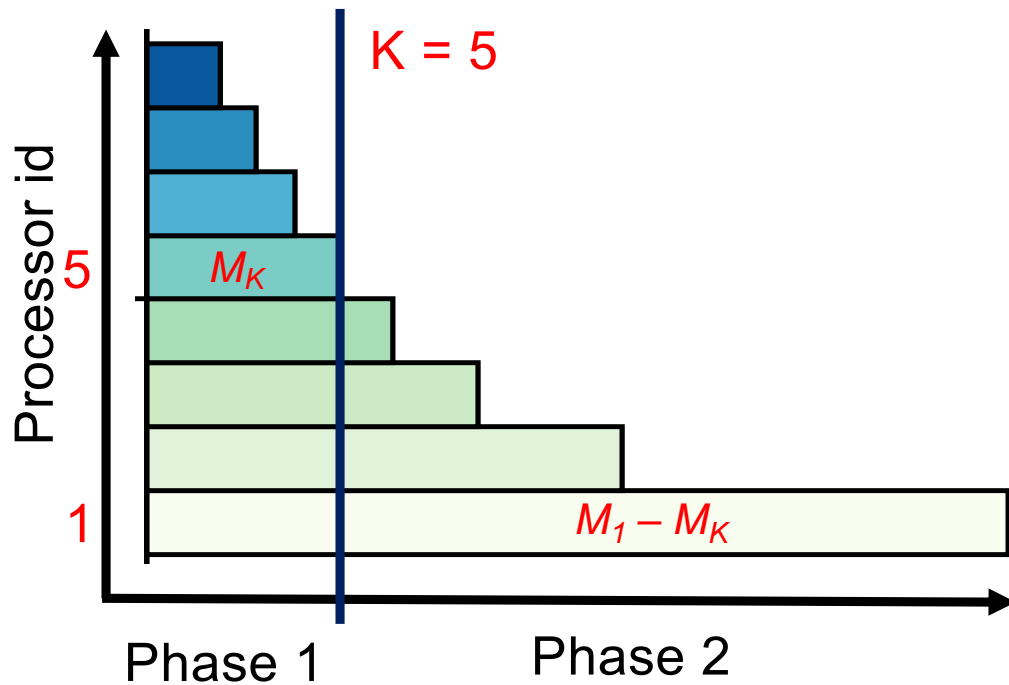
Split Computation in 2 Phases



Predicting Phase Duration



Predicting Phase Duration



$$T_1 = (\sum_{K < i \leq P} M_i + K M_K) / \rho \quad T_2 = (M_1 - M_K) / \beta$$

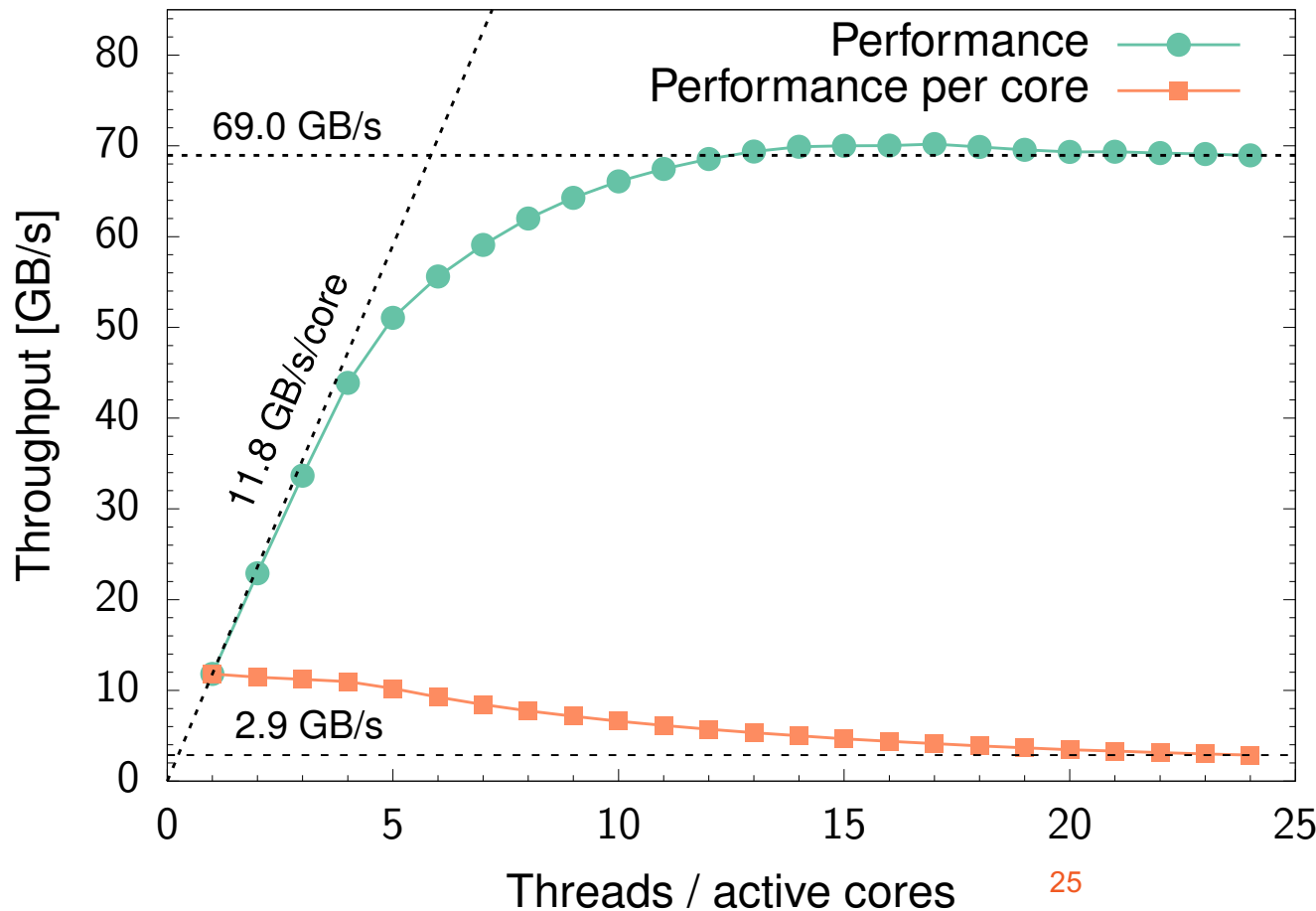
No matter how many processors are active:

- Phase 1 performance will be ρ
- Processor 1 performance will be β in Phase 2

simula

The 2 Phase Model

$$T = (\sum_{K < i \leq P} M_i + K M_K) / \rho + (M_1 - M_K) / \beta$$

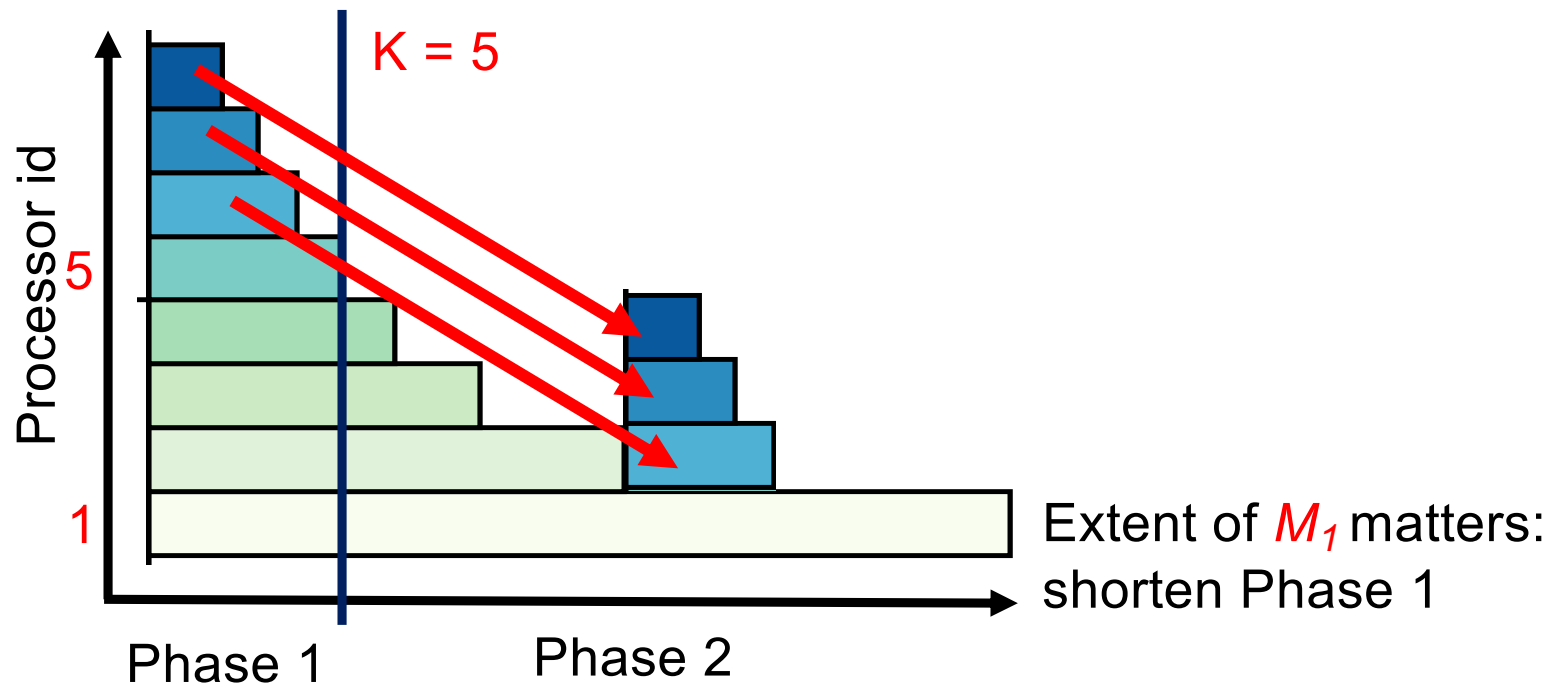


Real processors come close to the model, but phase transition is softer

24-core Intel Xeon
Platinum 8168 (Skylake)

simula

Immediate consequence: Fewer Cores can be faster



Volume in Phase 1 matters:
reduce volume by moving it
to Phase 2

Testing the Model: CPU Data

CPU	P	β	ρ	K
AMD Epyc 7302P (Rome)	16	22.83	90.91	5
AMD Epyc 7413 (Milan)	24	31.83	102.58	4
AMD Epyc 7601 (Naples)	32	18.15	85.42	5
AMD Epyc 7763 (Milan)	64	30.93	121.23	4
Intel Xeon Gold 6130	16	13.42	74.74	6
Intel Xeon Platinum 8168	24	11.81	68.96	6
Intel Xeon Platinum 8360Y	36	14.9	158.21	11
Cavium ThunderX2	32	15.51	118.54	8
HiSilicon Kunpeng 920	64	12.35	131.54	11
NVIDIA Grace GH200	72	27.16	316.45	12

AMD: low **K** value

Intel / ARM

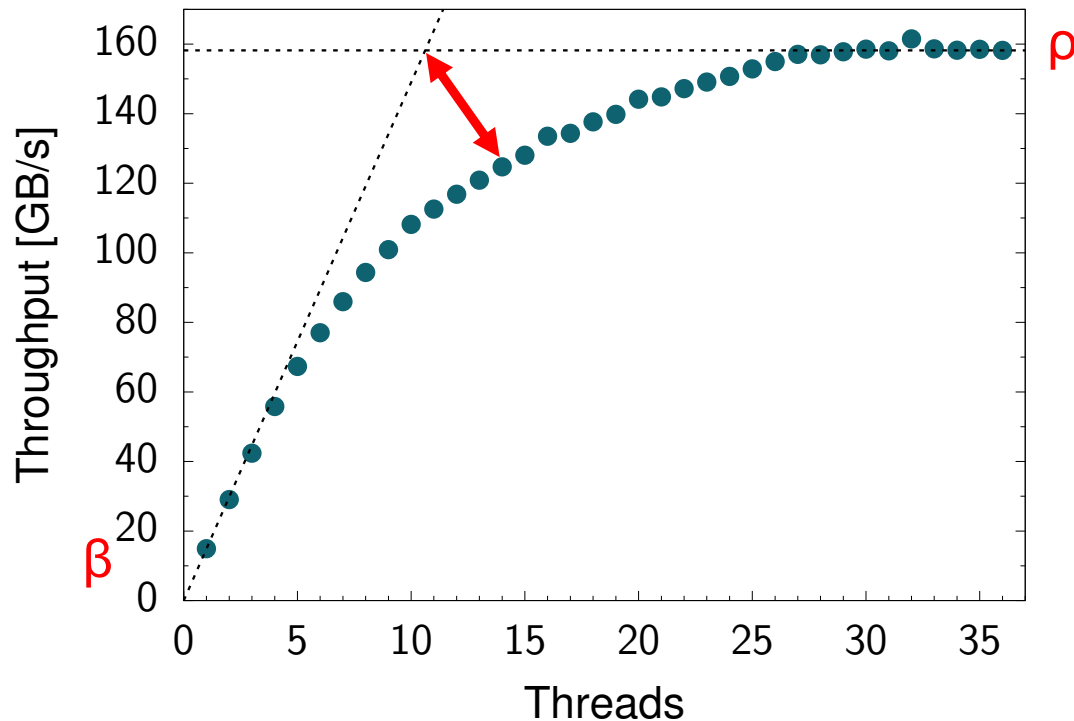
Medium to high **K** value

Does it work for Triangular Workloads?

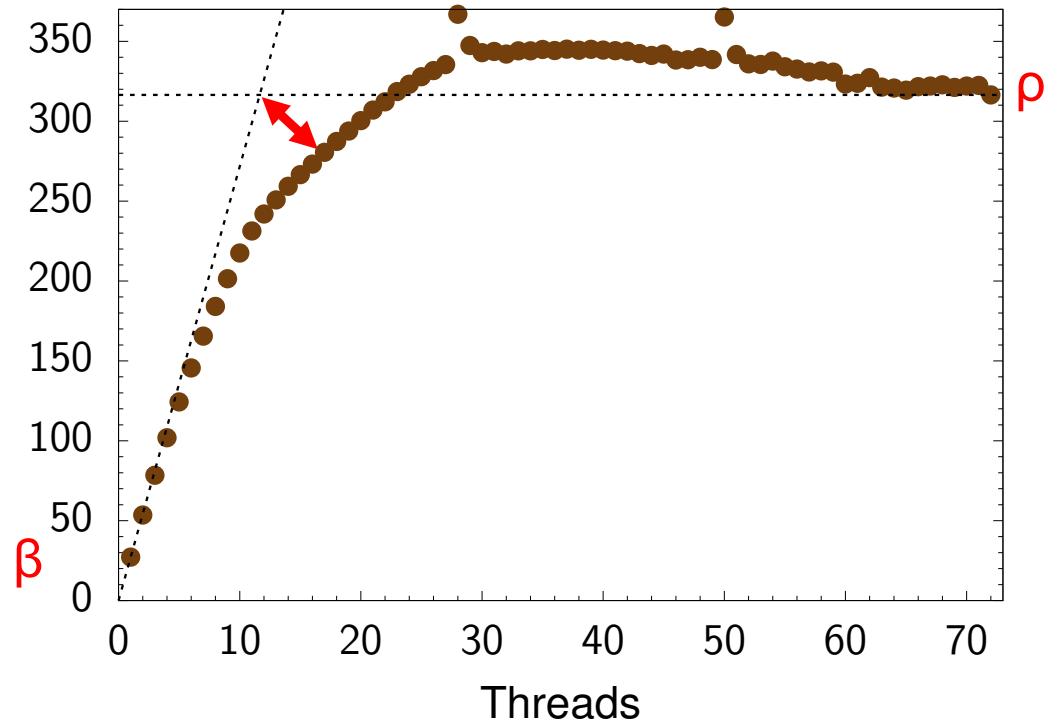
	Measured	Staircase Model		2-Phase Model	
CPU	[GB/s]	[GB/s]	Error	[GB/s]	Error
AMD Epyc 7302P (Rome)	60.24	57.26	-5%	83.16	41%
AMD Epyc 7413 (Milan)	88.21	91.06	3%	100.25	14%
AMD Epyc7601 (Naples)	66.9	72.1	8%	83.02	24%
AMD Epyc7763 (Milan)	127.07	117.41	-8%	120.71	-5%
Intel Xeon Gold 6130	44.1	50.55	15%	63.42	44%
Intel Xeon Platinum 8168	63.98	59.97	-6%	63.61	-1%
Intel Xeon Platinum 8360Y	89.67	129.36	44%	137.77	54%
Cavium Thunder X2	74.09	96.28	30%	108.67	47%
HiSilicon Kunpeng 920	92.46	102.17	10%	125.62	36%
NVIDIA Grace GH200	273.57	309.8	13%	302.77	11%

2-Phase is less accurate than Staircase, but much better than simple models

Where does the Difference come from?



Intel Xeon Platinum 8360Y (Ice Lake)



NVIDIA Grace GH200 CPU

CPU's with STREAM results closer to the intersection point can be predicted more accurately.

Does it work for Amdahl Workloads?

	Measured	2-Phase	Model
CPU	[GB/s]	[GB/s]	Error
AMD Epyc 7302P (Rome)	33.07	36.49	+10.35%
AMD Epyc 7413 (Milan)	43.52	48.58	+11.62%
AMD Epyc 7601 (Naples)	22.46	29.94	+33.29%
AMD Epyc 7763 (Milan)	47.34	49.28	+4.11%
Intel Xeon Gold 6130	22.07	22.75	+3.10%
Intel Xeon Platinum 8168	20.75	20.16	-2.84%
Intel Xeon Platinum 8360Y	28.14	27.24	-3.22%
Cavium ThunderX2	26.90	27.43	+1.99%
HiSilicon Kunpeng 920	22.21	22.58	+1.67%
NVIDIA Grace GH200	51.84	50.03	-3.49%

Low **K** value means
AMDs perform well here

- Accurate on all CPUs except Naples
- Identical to Staircase model here 30

Summary

- Load imbalance causes problems for performance models
- Sample imbalanced workloads are easy to assign
- New model built on Roofline and Max-rate models
- Easy to apply. Uses only P, β, ρ

Summary

- Load imbalance causes problems for performance models
- Sample imbalanced workloads are easy to assign
- New model built on Roofline and Max-rate models
- Easy to apply. Uses only P, β, ρ
- Model is fairly accurate, except for processor-specific behaviour
- Can be used for internode communication (multiple cores, one NIC)
- Main insight: Phase 1 depends on the total, Phase 2 only on M_1
- K and M_K determine phase transition, crucial for performance

Questions?